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# Observational Cosmology and the Cosmic Distance Duality Relation

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**Abstract.** We study the validity of cosmic distance duality relation between angular diameter and luminosity distances. To test this duality relation we use the latest Union2 Supernovae Type Ia (SNe Ia) data for estimating the luminosity distance. The estimation of angular diameter distance comes from the samples of galaxy clusters (real and mock) and FRIIb radio galaxies. We parameterize the distance duality relation as a function of redshift in six different ways. Our results rule out some of the parameterizations significantly.

**Keywords:** supernovae type Ia - standard candles, galaxy clusters, dark energy theory

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## 1 Introduction

To understand the physics behind the late time cosmic acceleration is one of the most challenging problem in the cosmology. Many observational methods using distant type Ia supernovae (SNe Ia), Cosmic Microwave Background (CMB), Galaxy Clusters, Baryon Acoustic Oscillations (BAO) etc. have been used to probe the mechanism behind this positive acceleration [1]. These tools are based on the fundamental assumption that total number of photons are conserved on cosmic scales. This is one of the key assumptions in the relationship between the luminosity distance,  $d_L$ , and the angular diameter distance,  $d_A$ , [2], i.e.

$$\frac{d_L}{d_A(1+z)^2} = 1. \quad (1.1)$$

This equation is known as Distance Duality (DD) relation [3]. It is natural to check the validity of this equation which may play a key role in observational cosmology. In particular it plays a vital role in galaxy observations, CMB observations and gravitational lensing [4]. Furthermore, violation in the DD relation may point to a failure of the metric theory of gravity in explaining the background dynamics of the universe and emergence of new physics. Whether this DD relation can shed some light on the presence of exotic physics or not was first explored by Bassett & Kunz (2004) [2]. They ruled out non-accelerating models of universe (replenishing dust model) by more than  $4 - \sigma$  level. In this context to study the validity of DD relation (1.1), we analyse the following red-shift dependence of DD

$$\eta(z) \equiv \frac{d_L}{d_A(1+z)^2}. \quad (1.2)$$

Uzan et al.(2004) used the combined measurements of Sunyaev-Zeldovich effect and X-ray emission data of galaxy clusters to study the violation in the DD relation [5]. They showed that if this relation does not hold then the angular diameter distance measured from the clusters is  $d_A^{cluster}(z) = d_A \eta^2$ , and hence the DD relation (1.2) gets modified to

$$\eta(z) \equiv \frac{d_A^{cluster}(1+z)^2}{d_L}. \quad (1.3)$$

They found no significant violation of DD relation as they obtained the value of  $\eta$  close to 1. Their data set consisted of 18 galaxy clusters. Further, De Bernardis et al.(2006) found no departure from DD relation i.e.  $\eta = 1$  at  $1\sigma$  C. L.[6]. They used bigger sample of 38

galaxy clusters in the redshift range of  $0.14 < z < 0.89$ . Later on Corasaniti (2006) modeled the intergalactic dust in terms of the star formation history of the universe and forecasted a deviation in the DD relation due to the presence of the cosmic dust extinction [7].

Recently Holanda et al.(2011) used two different data sets of galaxy clusters to check the consistency of DD relation [8]. These two data sets were based on two different geometrical descriptions of clusters. By assuming two parameterizations for  $\eta(z)$ , they showed that the data set which is based on the elliptical  $\beta$  model of clusters gives better fit with DD relation as compared to the data set based on spherical  $\beta$  model. Furthermore, Holanda et al. (2010) checked the validity of DD relation using the galaxy cluster sample and SNe Ia with different parameterizations of  $\eta(z)$  [9]. Hence they obtained bounds on the parameters of  $\eta(z)$ . They concluded that the best fit values of the parameters of the  $\eta(z)$  parametrization obtained through the data set based on spherical  $\beta$  model of cluster are not consistent with the DD relation.

Lazkoz et al. found no evidence of violation in this relation at  $2\sigma$  level [10]. They used SNe Ia and CMB + BAO as standard candles and standard rulers respectively to test the validity of this relation.

Further Avgoustidis et al. demonstrated that the DD relationship can be used to put constraints on the cosmic transparency [11]. They obtained the distances by using SNe Ia data, BAO and measurements of the Hubble parameter,  $H(z)$ . Following the same line of thought More et al. also explored the cosmic transparency (conservation of photons phase-space density) by using distance measures [12].

More recently Lampeitl et al. analyzed the DD relation and found no proof of violation at the one sigma level. For this analysis, they used SDSS II SNe data and local BAO measurements at redshift  $z= 0.2$  and  $0.35$  [13].

The aim of this paper is to reanalyze the validity of DD relation in a more comprehensive manner by using different data samples and parameterizations. In this work we have used three different data-sets of galaxy clusters, radio galaxies and a Mock data set to determine angular diameter distance, along with Union2 sample of SNe Ia. We have analyzed the validity of DD relation by using six different parameterizations of  $\eta(z)$ .

The plan of the paper is as follows. In the next section we describe various parameterizations considered in this work. The details of the data and methodology used is also explained in this section. We conclude with a section on results and discussion.

## 2 Parameterizations, data and method

### 2.1 $\eta(z)$ parameterizations

In this work we parameterized the  $\eta(z)$  with one index and two index parameterizations. These relations are inspired by model independent parameterizations for the dark energy equation of state [14]. We are parameterizing  $\eta(z)$  whose value stays one when photon number is conserved, gravity is described by a metric theory and photons travel on null geodesics. Any significant violation from the DD relation will hint at the emergence of new physics. Since all these assumptions are reasonably well tested [2], we expect  $\eta$  to stay close to unity. To model any departure from unity we parameterize  $\eta$  with six parametric representations for a possible redshift dependence of the distance duality relation:

- Two index parameterizations

$$\eta_I(z) = \eta_0 + \eta_1 z \quad (2.1)$$

$$\eta_{II}(z) = \eta_2 + \eta_3 \frac{z}{1+z} \quad (2.2)$$

$$\eta_{III}(z) = \eta_4 + \eta_5 \frac{z}{(1+z)^2} \quad (2.3)$$

$$\eta_{IV}(z) = \eta_6 - \eta_7 \ln(1+z) \quad (2.4)$$

- One index parameterizations

$$\eta_V(z) = \eta_8 \left( \frac{1}{1+z} \right) \quad (2.5)$$

$$\eta_{VI}(z) = \eta_9 \left( \frac{1}{1+z} \right) \exp \left( \frac{z}{1+z} \right) \quad (2.6)$$

Both, one and two index parameterizations have their limitations and advantages. The two index parameterizations are more flexible in comparison to one index parameterizations which may completely dominate over data.

## 2.2 Data

We have used three different data sets to calculate the angular diameter distance. For the luminosity distances, we use latest Union2 SNe Ia data [15]. Now we consider that pair of galaxy cluster/radio galaxy and SNe for which  $\Delta z < 0.005$  [9]. Because of this condition, the number of data points are limited to 24, 222 and 12 for the data set I, II and III respectively. If there are more than one supernova satisfying  $\Delta z < 0.005$  for a given cluster/radio galaxy we choose the nearest SNe to the given cluster/radio galaxy. In case of multiple SNe at same redshift value for a given cluster/radio galaxy we choose that SNe which has less error bars.

- Data Set I: This sample consists of 25 galaxy clusters (isothermal elliptical  $\beta$  model, with concordance model for the cosmological distance-redshift relationship) [16]. By combining together the Sunyaev-Zeldovich temperature decrements and X-ray surface brightness observations, one can obtain the angular diameter distance for clusters. The redshift interval for clusters in this sample is  $0.02 < z < 0.78$ . After applying the selection criterion, the number of data points reduce to 24 in this sample.
- Data Set II: This bigger data set contains 578 mock values of angular diameter distances of mock clusters, and for 222 mock clusters we have SNe Ia satisfying  $\Delta z < 0.005$ . This catalog is created by assuming fiducial cosmology from the *WMAP7* + *BAO* +  $H_0$  results [17]. This data set assumes spherical isothermal  $\beta$  model for Intra-cluster medium (ICM). This mock catalog is created for overlap of ACT/SPT + eROSITA with 25% Gaussian scatter error which are similar to the recent measurements of angular diameter distance [18]. The estimate of angular diameter distances strongly depends on the ICM model. The redshift distribution of this sample is from  $0.05 < z < 0.76$ . For more details see the ref.'s [19, 20].

- Data Set III: In this sample the calculation of angular diameter distance is obtained by using the physical size of extended radio galaxies [21]. This data set contains 20 radio galaxies up to redshift  $z = 1.8$ . For 12 galaxies we could find corresponding SNe Ia where the condition  $\Delta z < 0.005$  is satisfied.

### 2.3 Method

We perform the  $\chi^2$  analysis to fit the parameters of the assumed parameterizations.

$$\chi^2(p) = \sum_i \frac{(\eta_{th}(z_i, p) - \eta_{obs}(z_i))^2}{\sigma(z_i)^2}. \quad (2.7)$$

Where  $\eta_{th}$  is the assumed form of parameterizations given by eq.(2.1) to eq.(2.6) and  $\eta_{obs}$  is the observed value of  $\eta$ , which is calculated by using  $d_L$  and  $d_A$  at a particular value of redshift. The unknown parameters in the parameterizations are denoted by the variable  $p$ . To draw the likelihood contours at 1, 2 and  $3\sigma$ ,  $\Delta\chi^2 = \chi^2 - \chi_{min}^2 = 2.3, 6.17$  and  $11.8$  respectively in two dimensional parametric space. For one parameter fitting,  $\Delta\chi^2 = 1, 4$  and  $9$  for 1, 2 and  $3\sigma$  C.L. respectively.

In order to find  $\eta_{obs}$  for cluster, we need the observed value of  $d_A^{cluster}$  and  $d_L$  as mentioned in eq. (1.3). The luminosity distance is obtained by using the latest Union2 SNe Ia data. The angular diameter distance,  $d_A^{cluster}$ , is obtained directly by using the data set I and II. While calculating the error bars on  $\eta_{obs}$ , we only considered the errors in the angular diameter distance values. The errors in the  $d_L$  values are quite small compared to the errors in  $d_A$ , and hence are neglected [9]. We know that the variance of a dependent variable  $f(x_j)$ , that depends on, say,  $n$  number of independent variables  $x_j$ , is given by the error propagation equation as

$$\sigma_f^2 = \sum_j \left[ \sigma_{x_j}^2 \left( \frac{\partial f}{\partial x_j} \right)^2 \right]. \quad (2.8)$$

Hence, the error in  $\eta_{obs}$  for galaxy clusters (mock and real) data set is

$$\sigma_{\eta_{obs}(i)} = \sigma_{d_A(i)} \frac{(1 + z_i)^2}{d_L(i)}. \quad (2.9)$$

Similarly the  $\eta_{obs}$  for the radio galaxies can be obtained by assuming  $d_A^{radio} = d_A$  (R. A. Daly, private communication) in eq. (1.2). The angular diameter distance for radio galaxies is given in terms of the dimensionless coordinate distance,  $y$ , as

$$d_A(i) = \frac{y(z_i)}{H_0(1 + z_i)}, \quad (2.10)$$

where  $y = a_0 r H_0$ , and  $a_0 r$  is the coordinate distance [21]. Treating Hubble constant,  $H_0$ , as a nuisance parameter we marginalize over it by assuming gaussian prior,  $H_0 = 74.2 \pm 3.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$  [22]. Finally, the uncertainty on the calculated value of  $\eta_{obs}$ , for radio galaxies can be obtained by using the formula

$$\sigma_{\eta_{obs}(i)} = \sigma_{d_A(i)} \frac{d_L(i)}{[d_A(i) (1 + z_i)]^2}. \quad (2.11)$$

In this analysis distances are obtained by assuming the flat  $\Lambda$  CDM universe.

### 3 Results and Discussions

In this work, we study the validity of DD relation using different data sets. We assume six general parameterizations of  $\eta(z)$  completely in analogy with the varying equation of state of dark energy,  $\omega(z)$ . In these different parameterizations it is assumed that  $\eta(z)$  evolves with redshift.

Our results are summarized as follows:

1. We find the best fit values of the parameters in all the six parameterizations with data set I. This data set contains angular diameter of 24 galaxy clusters. Using  $\chi^2$  minimization technique we obtain the best fit values of the parameters within  $1\sigma$  range shown in Table 1.

**Table 1.** Best fit values for all parameterizations - data set I

$\chi^2_\nu$	Parameters	
1.217	$\eta_0 = 0.999 \pm 0.144$	$\eta_1 = -0.058 \pm 0.507$
1.216	$\eta_2 = 1.007 \pm 0.170$	$\eta_3 = -0.118 \pm 0.822$
1.216	$\eta_4 = 1.013 \pm 0.205$	$\eta_5 = -0.198 \pm 1.345$
1.216	$\eta_6 = 1.003 \pm 0.157$	$\eta_7 = 0.087 \pm 0.651$
1.337	$\eta_8 = 1.194 \pm 0.061$	
1.169	$\eta_9 = 1.01 \pm 0.051$	

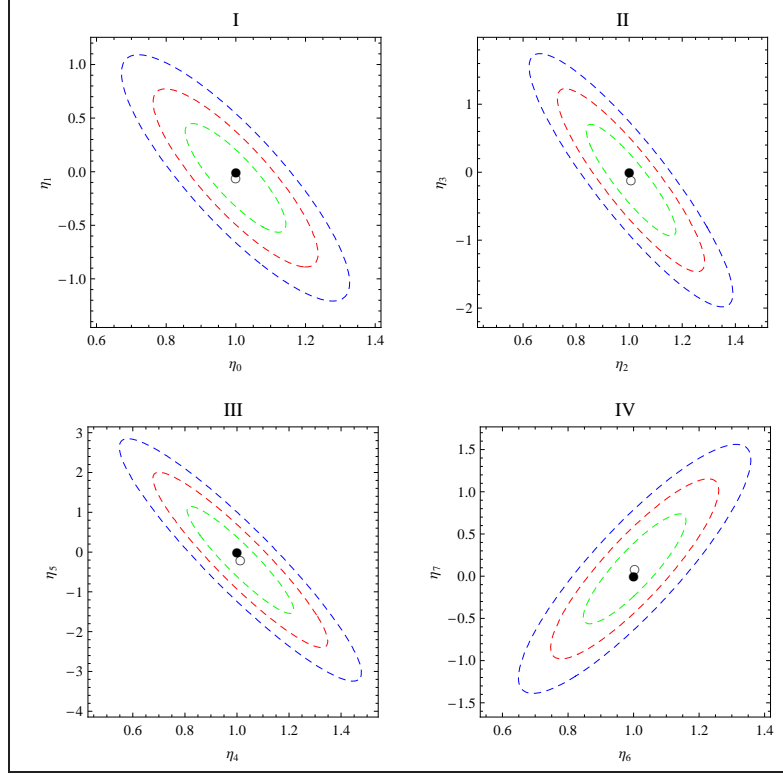
Here  $\chi^2_\nu$  is reduced- $\chi^2$ , or  $\chi^2$  per degree of freedom. All the two index parameterizations do support DD relation with in  $1\sigma$  C.L. as shown in Fig. 1 for data set I. In one index parameterizations,  $\eta_V(z)$ , shows significant deviation from DD relation, where as  $\eta_{VI}(z)$  is in agreement with the DD relation (see Fig. 2).

2. The data set II which contains the mock catalog of angular diameter distances of galaxy clusters is the biggest data used for this purpose so far. It consists of 222 galaxy clusters after using the selection criterion ( $\Delta z < 0.005$ ). The best fit values for all the parameterizations with in  $1\sigma$  are shown in Table 2.

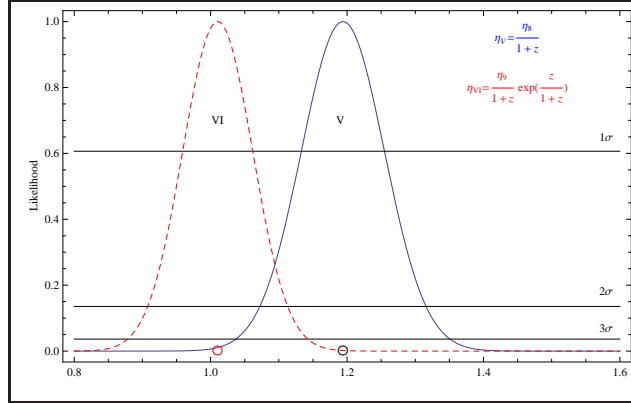
**Table 2.** Best fit values for all parameterizations - data set II

$\chi^2_\nu$	Parameters	
1.076	$\eta_0 = 0.973 \pm 0.048$	$\eta_1 = -0.108 \pm 0.159$
1.078	$\eta_2 = 0.971 \pm 0.058$	$\eta_3 = -0.138 \pm 0.267$
1.080	$\eta_4 = 0.972 \pm 0.072$	$\eta_5 = -0.141 \pm 0.453$
1.077	$\eta_6 = 0.972 \pm 0.053$	$\eta_7 = 0.124 \pm 0.208$
1.228	$\eta_8 = 1.163 \pm 0.019$	
1.070	$\eta_9 = 0.972 \pm 0.016$	

As shown in Fig. 3, all two index parametrization show deviation from DD relation at  $3\sigma$  level. While Fig. 4, which shows the likelihood plot for  $\eta_V$  and  $\eta_{VI}$ , again indicates that in case of  $\eta_V$  there is a significant deviation from DD relation. Similarly  $\eta_{VI}$  only marginally accommodates the distance duality relation. Thus, the one index parameterizations also do not support the DD relation convincingly. It is important



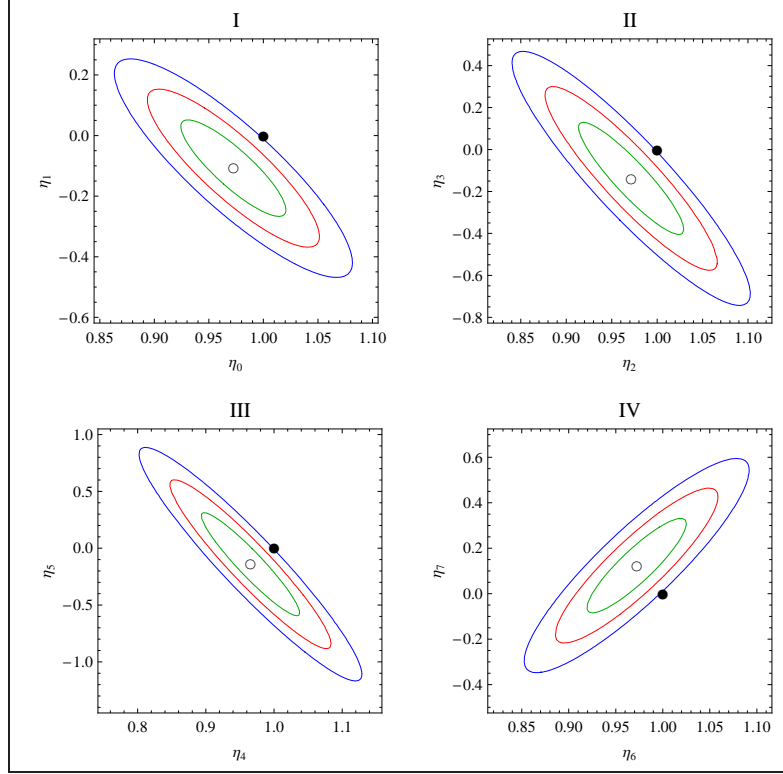
**Figure 1.**  $1\sigma$ ,  $2\sigma$  and  $3\sigma$  contours in  $\eta_i - \eta_j$  plane with data set I. The position of filled circle in the contours indicate the point where  $\eta(z = 0) = 1$ . The position of empty circle indicates the best fit value of the parameters.



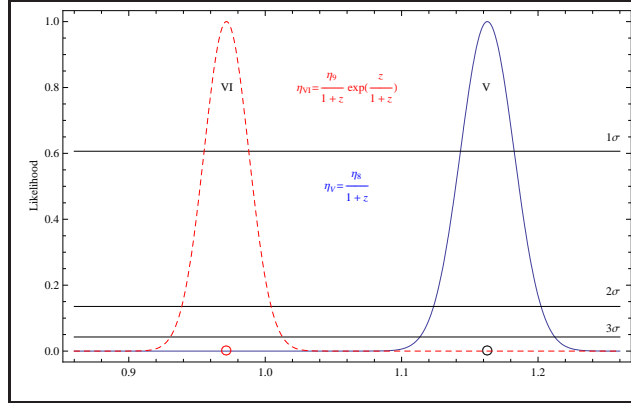
**Figure 2.** Likelihood distribution function from the data Set I. The position of empty circle indicates the best fit value of the parameters

to note that five out of six parameterizations do not give a substantial support to the DD relation with this bigger mock data set. It may indicate that the underlying assumptions used to create this mock data set need revision.

3. In the data set III, the angular diameter distance is obtained by using the extended physical size of radio galaxies. After using the selection criterion ( $\Delta z < 0.005$ ) we are left with 12 data points in this set. The best fit values of the parameters using  $\chi^2$



**Figure 3.**  $1\sigma, 2\sigma$  and  $3\sigma$  contours in  $\eta_i - \eta_j$  plane with data set II. The position of filled circle in the contours indicate the point where  $\eta(z=0) = 1$ . The position of empty circle indicates the best fit value of the parameters.



**Figure 4.** Likelihood distribution from Mock data sample of galaxy clusters ( data Set II ). The position of empty circle indicate the best fit value of the parameters.

minimum method at  $1\sigma$  level are given in Table 3.

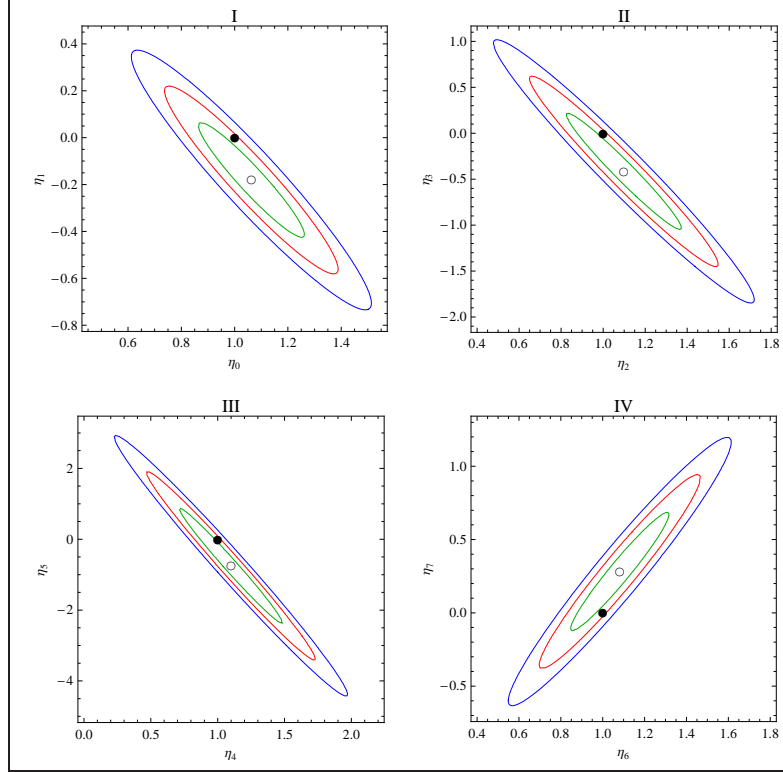
As shown in Fig. 5 and 6, this data set does not support DD relation for both 2 index and 1 index parameterizations convincingly.

4. It is important to note that all two index parameterizations show degenerate behaviour with the given data sets. These parameterizations are in complete concordance with



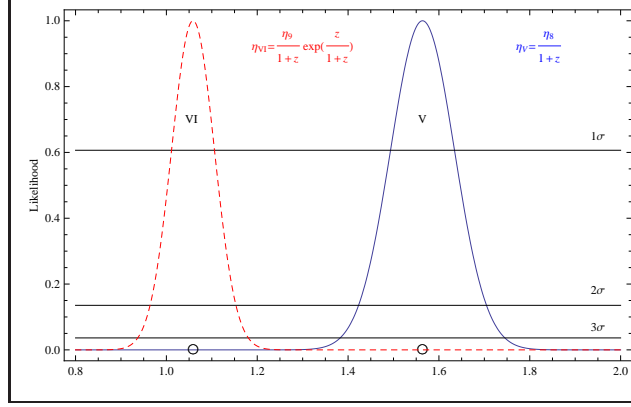
**Table 3.** Best fit values for all parameterizations - data set III

$\chi^2_\nu$	Parameters	Parameters
0.944	$\eta_0 = 1.063 \pm 0.198$	$\eta_1 = -0.180 \pm 0.244$
0.971	$\eta_2 = 1.099 \pm 0.274$	$\eta_3 = -0.415 \pm 0.632$
1.021	$\eta_4 = 1.099 \pm 0.385$	$\eta_5 = -0.749 \pm 1.624$
0.958	$\eta_6 = 1.081 \pm 0.235$	$\eta_7 = 0.282 \pm 0.404$
1.592	$\eta_8 = 1.564 \pm 0.070$	
0.870	$\eta_9 = 1.059 \pm 0.047$	

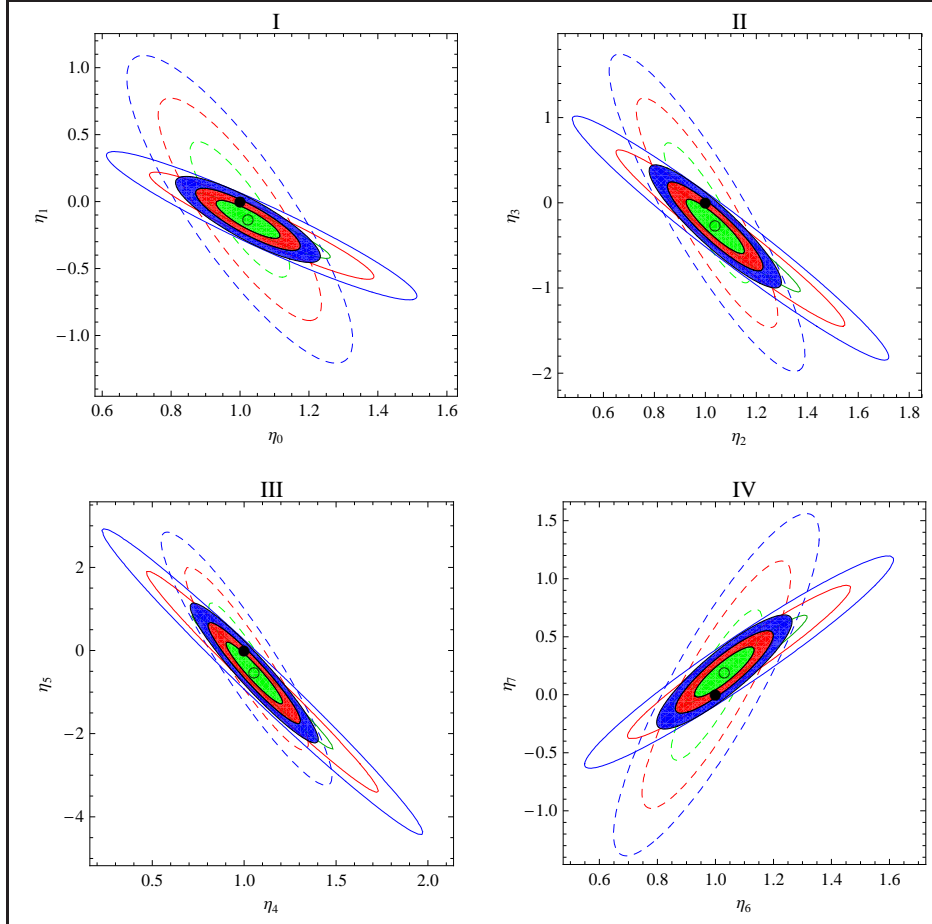
**Figure 5.**  $1\sigma$ ,  $2\sigma$  and  $3\sigma$  contours in  $\eta_i - \eta_j$  plane with data set III. The position of filled circle in the contours indicate the point where  $\eta(z=0) = 1$ . The position of empty circle indicates the best fit value of the parameters.

DD relation within  $1\sigma$  level for data set I, shows deviation at  $3\sigma$  level for data set II, and in agreement with DD relation within  $2\sigma$  level for data set III. So the behaviour of all two index  $\eta(z)$ 's strongly depends upon the data sets chosen here.

5. The  $\chi^2_\nu$  for  $\eta_V$  parametrization with data sets I, II and III is approximately 1.34, 1.23 and 1.59, respectively. Hence the one index parametrization,  $\eta_V$ , shows significant deviation with all the three data sets. The  $\eta_{VI}$  parametrization marginally accommodates the DD relation with data set II and III, and convincingly accommodates data set I. It is evident that for one index parameterizations the chosen parametrization prior dominates over the data sets. Therefore the one index parameterizations analysed here are not a good choice for analysing the DD relation. This is in contrast with the be-

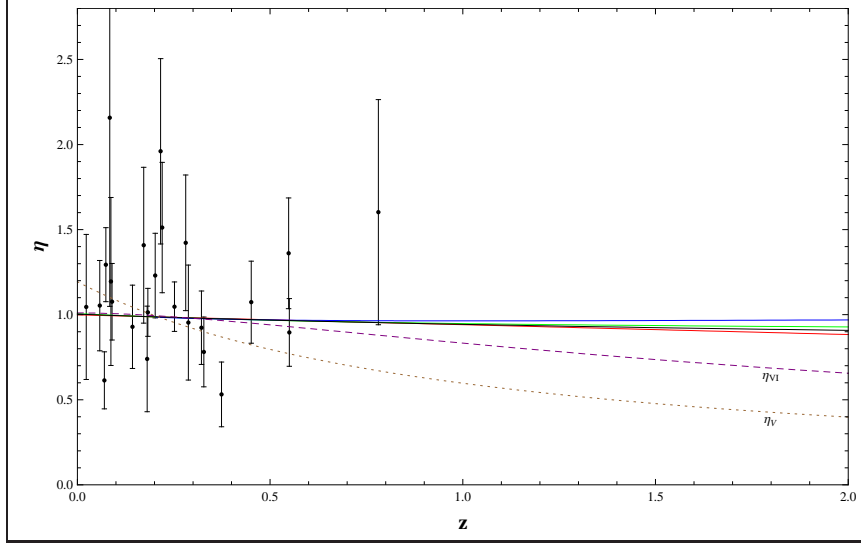


**Figure 6.** Likelihood distribution function from data set III. The position of empty circle indicate the best fit value of the parameters.

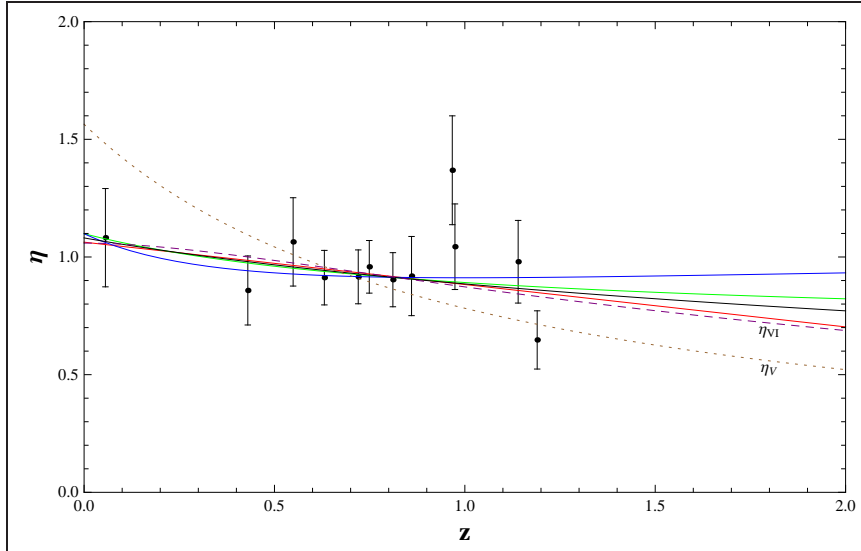


**Figure 7.** Dashed, solid and filled contours correspond to data set I, data set III and combined data set (I + III) respectively.

haviour shown by two index parameterizations in which data seems to dominate. To understand behaviour of the parameterizations in a qualitative manner we plot them using their best fit values along with real data sets (Figs. 8 and 9).



**Figure 8.** Variation of parameterizations with redshift using best fit values given in Table 1 . The points with error bars are from data set I. Solid line correspond to two index parameterizations. Dot and dashed lines correspond to one index parameterizations.



**Figure 9.** Variation of parameterizations with redshift using best fit values given in Table 3. The points with error bars are from data set III. Solid lines correspond to two index parameterizations. Dot and dashed lines correspond to one index parameterizations.

It is clear from the figures 8 and 9 that parametrization  $\eta_V(z)$  is not a good choice for modeling the DD relation. It shows a very steep deviation from 1 even at very low redshift ( $z \sim 0$ ), where we expect DD relation to hold. Hence  $\eta_V$  parametrization is clearly dominating over the data sets. But on the other side all the two index parameterizations stay close to unity (see Figs. 8 and 9).

6. Out of these three data sets, as expected, the bigger data set of mock galaxy clusters (data set II) gives tighter constraints on various parameterizations. But most of the

parameterizations show significant deviation from DD relation. It is important to note that the mock galaxy clusters data is generated by assuming the spherical isothermal  $\beta$  model for clusters. But the real galaxy cluster data (data set I) which is obtained by assuming isothermal elliptical  $\beta$  for clusters is in good agreement with DD relation.

7. The main advantage of using the radio galaxy data is shown in Fig. 7 where the contours (dashed lines) correspond to radio galaxies are aligned to contours corresponding to clusters (solid line). We also show the contours by using the combined data set of real galaxy clusters and radio galaxies. As expected the constraints on the parameters are improved.
8. Overall, the pattern seen between one and two-parameter parameterizations suggest that for the two-parameter case the data dominates the result, whereas for the one-parameter case the chosen parameterization prior dominates the result.

When we finished this work, we came across a paper by Li et al. who also have checked the violation of DD [23]. However, we have used more data sets, and we have worked with six parameterizations.

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